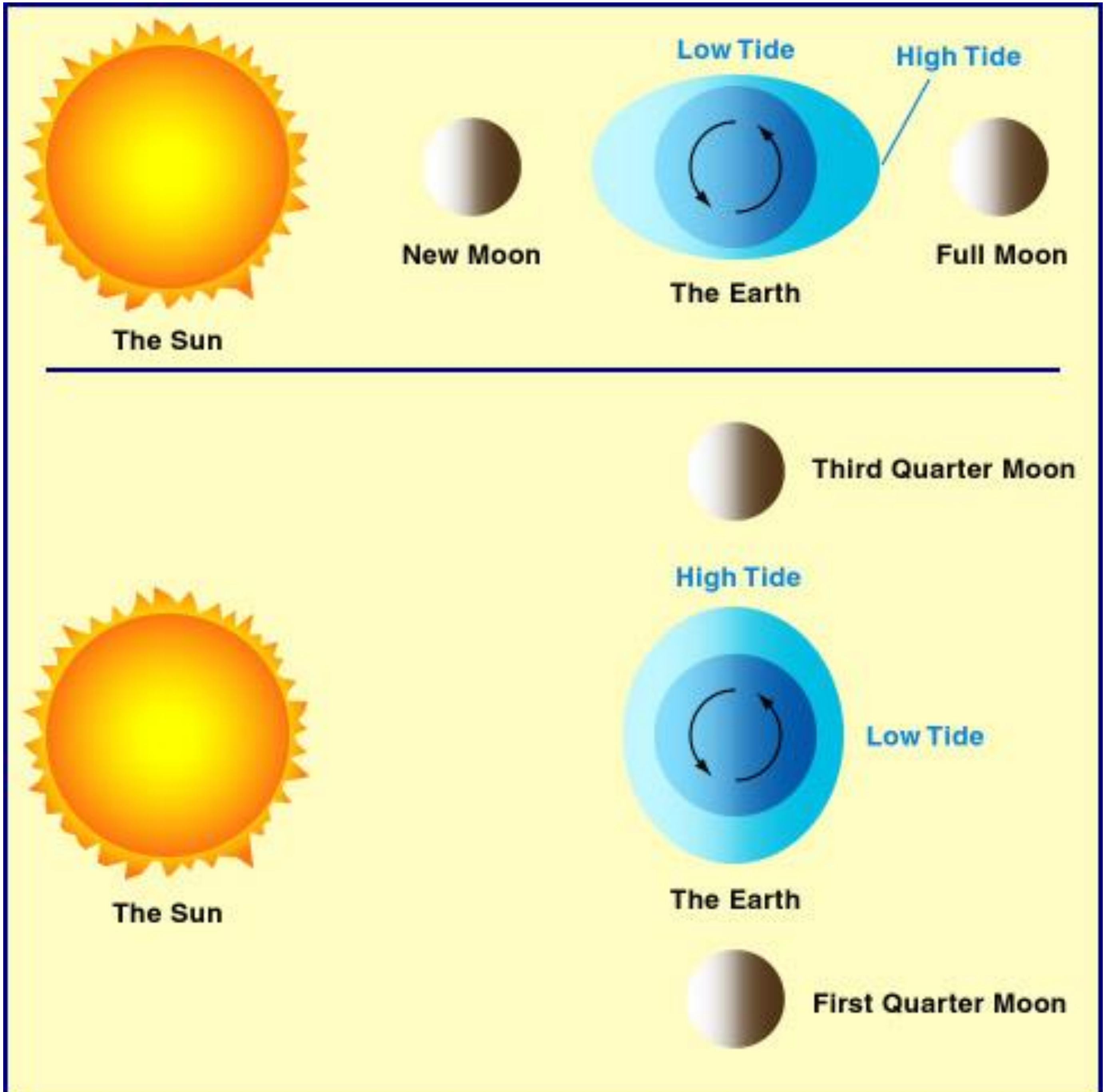
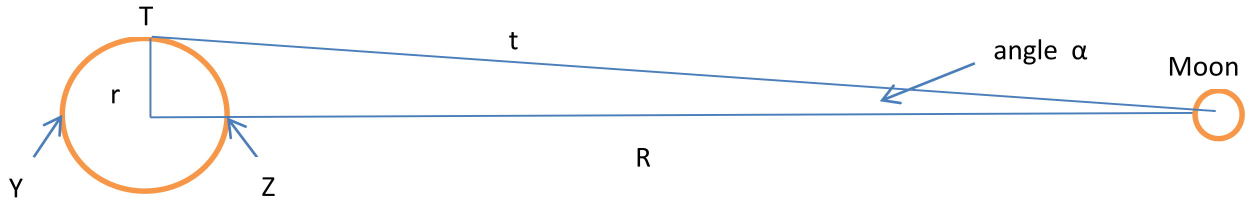

Low Tide
High Tide
New Moon
The Earth
Full Moon
The Sun
Third Quarter Moon
High Tide
Low Tide
The Sun
The Earth
First Quarter Moon




Earth, on left, has radius r, and its centre is a distance R from the centre of the Moon.

At the centre of the Earth, the gravitational acceleration owing to the Moon (mass M) is:

$$\frac{GM}{R^2}$$

At Z, it is

$$\frac{GM}{(R-r)^2}$$

At Y, it is

$$\frac{GM}{(R+r)^2}$$

At Z, the near-side tidal acceleration, in the direction **away** from the centre of the Earth, is:

$$\frac{GM}{(R-r)^2} - \frac{GM}{R^2}$$

At Y, the far-side tidal acceleration, **away** from the centre of the Earth and so in the opposite direction, is:

$$-\frac{GM}{(R+r)^2} - \left(-\frac{GM}{R^2}\right)$$

or

$$\frac{GM}{R^2} - \frac{GM}{(R+r)^2}$$

The result at Z rearranges to:

$$\frac{GM}{R^2}\left(\frac{1}{\left(1-\frac{r}{R}\right)^2} - 1\right)$$

The result at Y rearranges to:

$$\frac{GM}{R^2}\left(1 - \frac{1}{\left(1+\frac{r}{R}\right)^2}\right)$$

$\left(\frac{r}{R}\right)$ is much less than 1.

If you know the binomial theorem, then by inspection:

$$\left(\frac{1}{\left(1-\frac{r}{R}\right)^2}-1\right)\approx\frac{2r}{R}$$

and

$$\left(1-\frac{1}{\left(1+\frac{r}{R}\right)^2}\right)\approx\frac{2r}{R}$$

So the tidal acceleration term in both cases is :

$$\frac{GM}{R^2}\cdot\left(\frac{2r}{R}\right)=\frac{2GMr}{R^3}$$

## Avoiding the binomial theorem

$$\left(\frac{1}{\left(1-\frac{r}{R}\right)^2}-1\right)=\left(\frac{1}{1-\frac{2r}{R}+\left(\frac{r}{R}\right)^2}-1\right)\approx\left(\frac{1}{\left(1-\frac{2r}{R}\right)}-1\right)$$

If $x$ is much less than 1, then $\quad 1 \approx 1-x^2=(1+x)(1-x)$

So $\quad 1/(1-x)\approx(1+x)$

Applying this:

$$\left(\frac{1}{\left(1-\frac{2r}{R}\right)}-1\right)\approx\left(\left(1+\frac{2r}{R}\right)-1\right)=\frac{2r}{R}$$

Similarly

$$\left(1-\frac{1}{\left(1+\frac{r}{R}\right)^2}\right)\approx\left(1-\frac{1}{\left(1+\frac{2r}{R}\right)}\right)\approx\left(1-\left(1-\frac{2r}{R}\right)\right)=\frac{2r}{R}$$

$$\left(1 - \frac{1}{\left(1 + \frac{2r}{R}\right)}\right) \approx \left(1 - \left(1 - \frac{2r}{R}\right)\right) = \frac{2r}{R}$$

## Analogy with Simple Interest:

Principal = $\mathrm{P}$, annual interest rate = $i$, amount after one year = $\mathrm{A}$

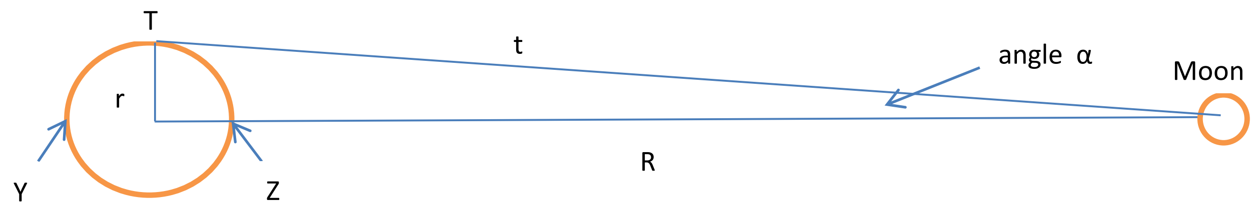
Interest after one year = $\mathrm{P}.i$

$\mathrm{A} = \mathrm{P} + \mathrm{P}.i = \mathrm{P}\,(1+i)$

So, $\mathrm{P} = \frac{A}{(1+i)}$

Suppose you only know the amount, and not the principal:

Interest after one year = $\mathrm{A} - P = \mathrm{A} - \frac{A}{(1+i)} = \mathrm{A}\left(1 - \frac{1}{(1+i)}\right) \approx \mathrm{A}.i$




Earth, on left, has radius r, and its centre is a distance R from the centre of the Moon.

At the centre of the Earth, the gravitational acceleration owing to the Moon is:

$$\frac{GM}{R^2}$$

At T, the acceleration can be resolved into two components:

Parallel to the Earth-Moon line: $\frac{GM}{t^2}.\cos\alpha$

Towards the centre of the Earth, perpendicular to the Earth-Moon line : $\frac{GM}{t^2}.\sin\alpha$

Pythagoras on the right-angled triangle: $t^{\mathbf{2}} = R^{\mathbf{2}} + r^{\mathbf{2}} = R^2\left(1 + \left(\frac{r}{R}\right)^{\mathbf{2}}\right)$

So $t = R\left(1 + \left(\frac{r}{R}\right)^{\mathbf{2}}\right)^{\frac{1}{2}}$

and $cos\,\alpha = \frac{R}{t} = \frac{1}{\left(1+\left(\frac{r}{R}\right)^2\right)^{\frac{1}{2}}}$ $sin\,\alpha = \frac{r}{t} = \frac{r}{R\left(1+\left(\frac{r}{R}\right)^2\right)^{\frac{1}{2}}}$

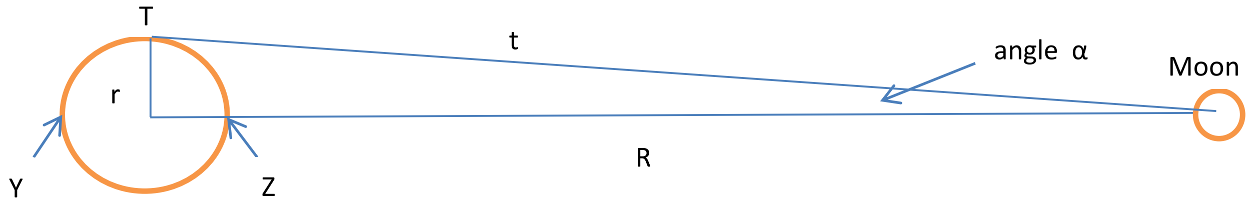
Component **parallel to the Earth-Moon line** is :

$$\frac{GM}{t^2}.\cos\alpha = \frac{GM}{R^2\left(1+\left(\frac{r}{R}\right)^{\mathbf{2}}\right)}.\frac{1}{\left(1+\left(\frac{r}{R}\right)^2\right)^{\frac{1}{2}}} = \frac{GM}{R^2\left(1+\left(\frac{r}{R}\right)^{\mathbf{2}}\right)^{\frac{3}{2}}} \approx \frac{GM}{R^2}$$

It matches that at Earth's centre

Component **perpendicular to the Earth-Moon line** is:

$$\frac{GM}{t^2}.\sin\alpha = \frac{GM}{R^2\left(1+\left(\frac{r}{R}\right)^{\mathbf{2}}\right)}.\frac{r}{R\left(1+\left(\frac{r}{R}\right)^2\right)^{\frac{1}{2}}} = \frac{GMr}{R^3\left(1+\left(\frac{r}{R}\right)^{\mathbf{2}}\right)^{\frac{3}{2}}} \approx \frac{GMr}{R^3}$$ and is wholly tidal

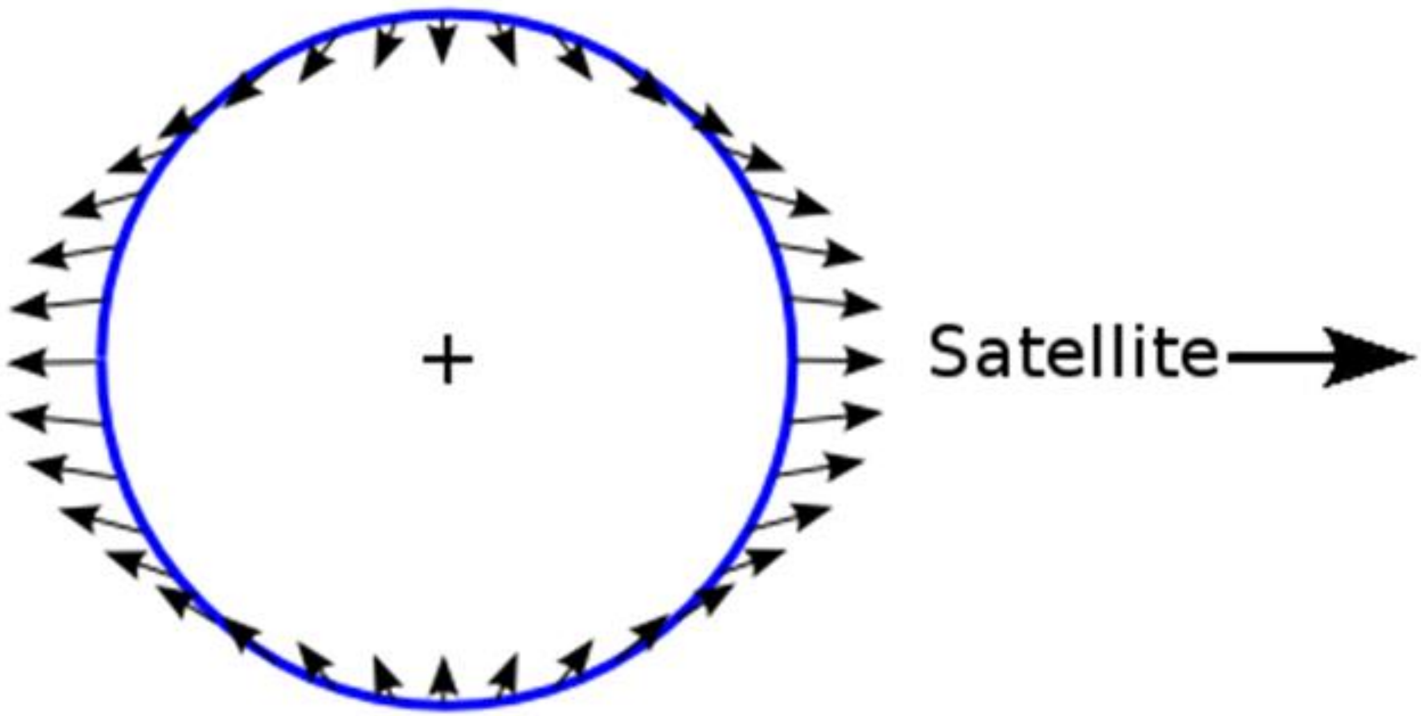


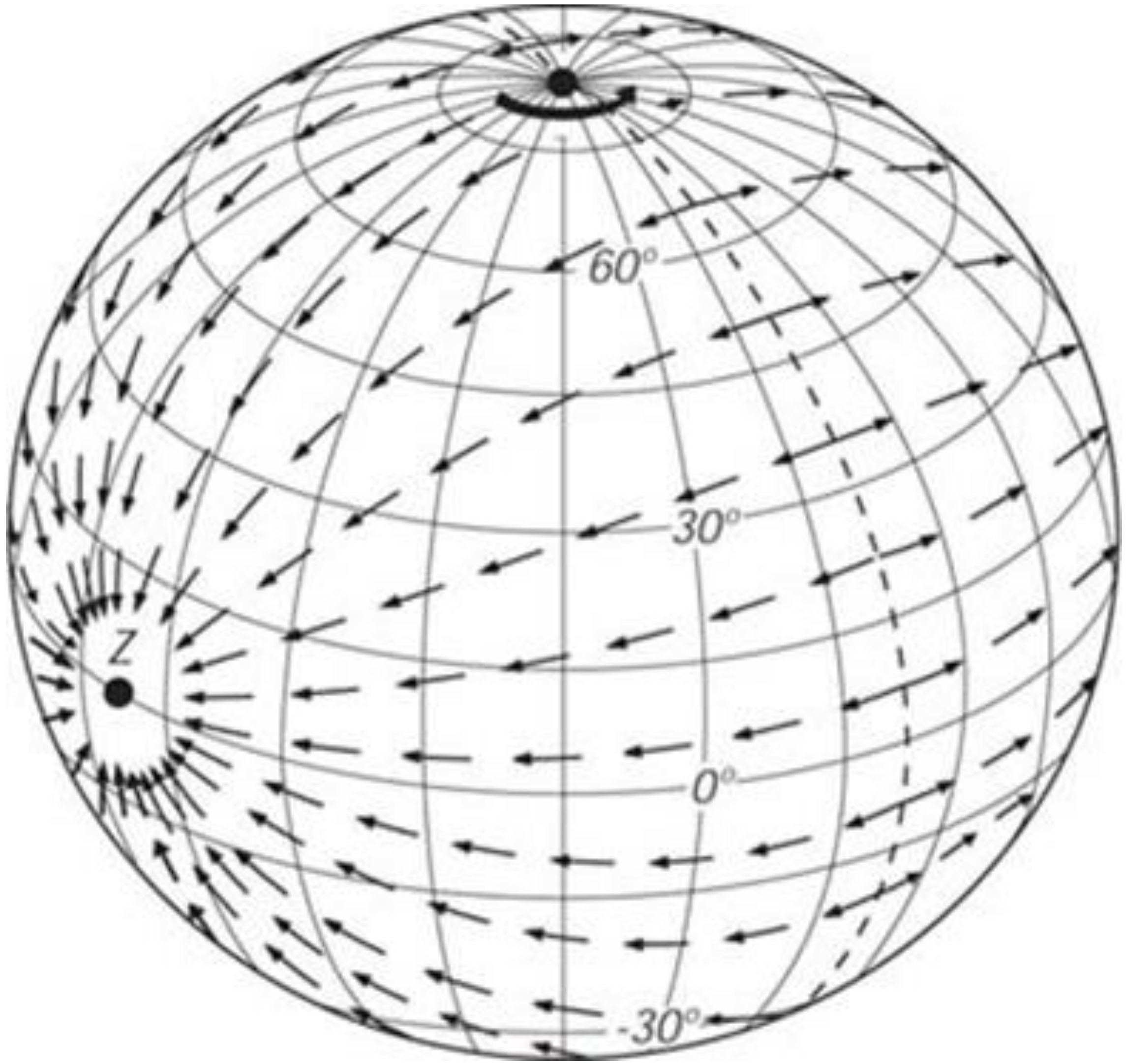

Earth, on left, has radius r, and its centre is a distance R from the centre of the Moon.

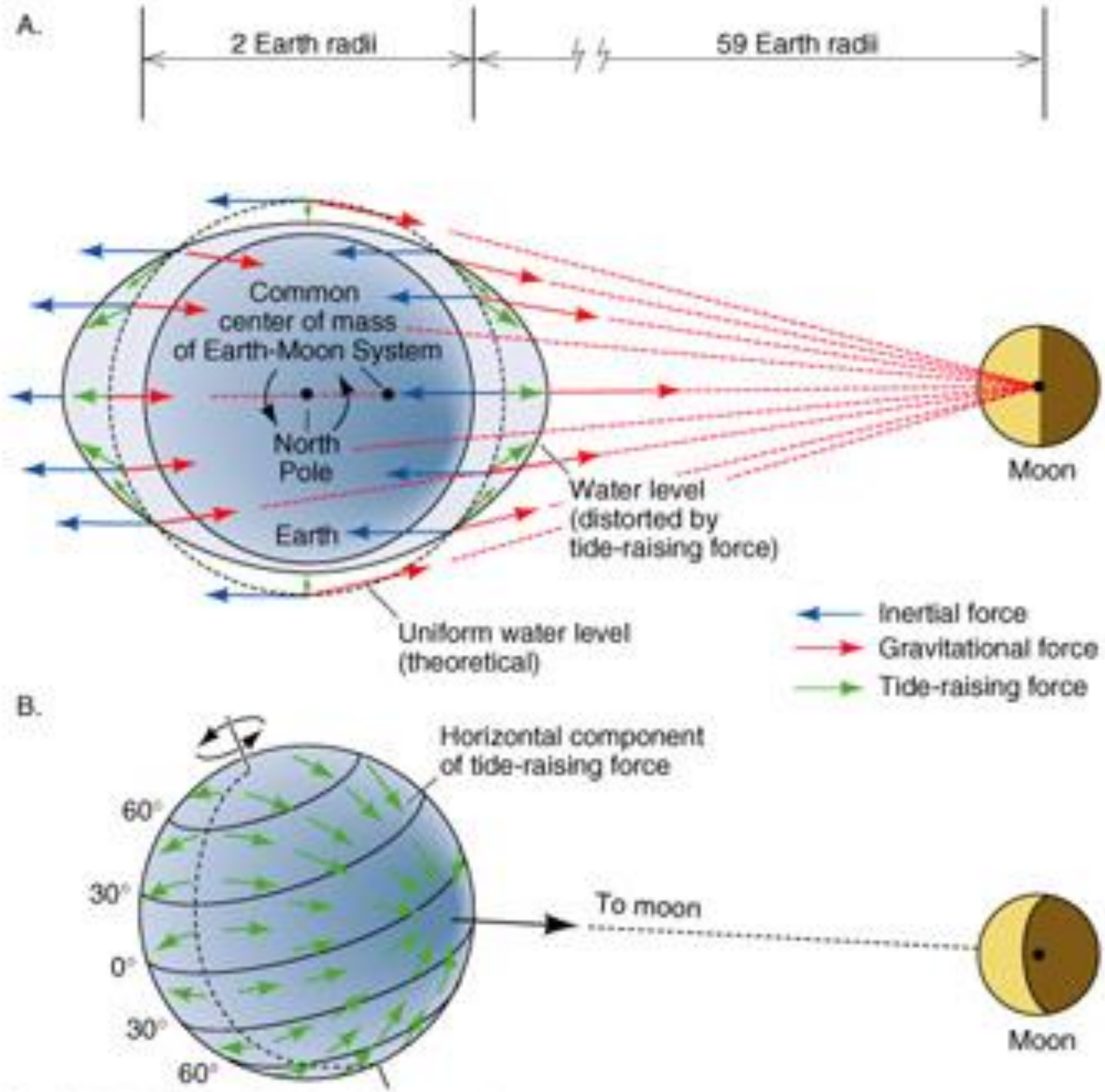
So, tidal acceleration at key points on earth's surface are:

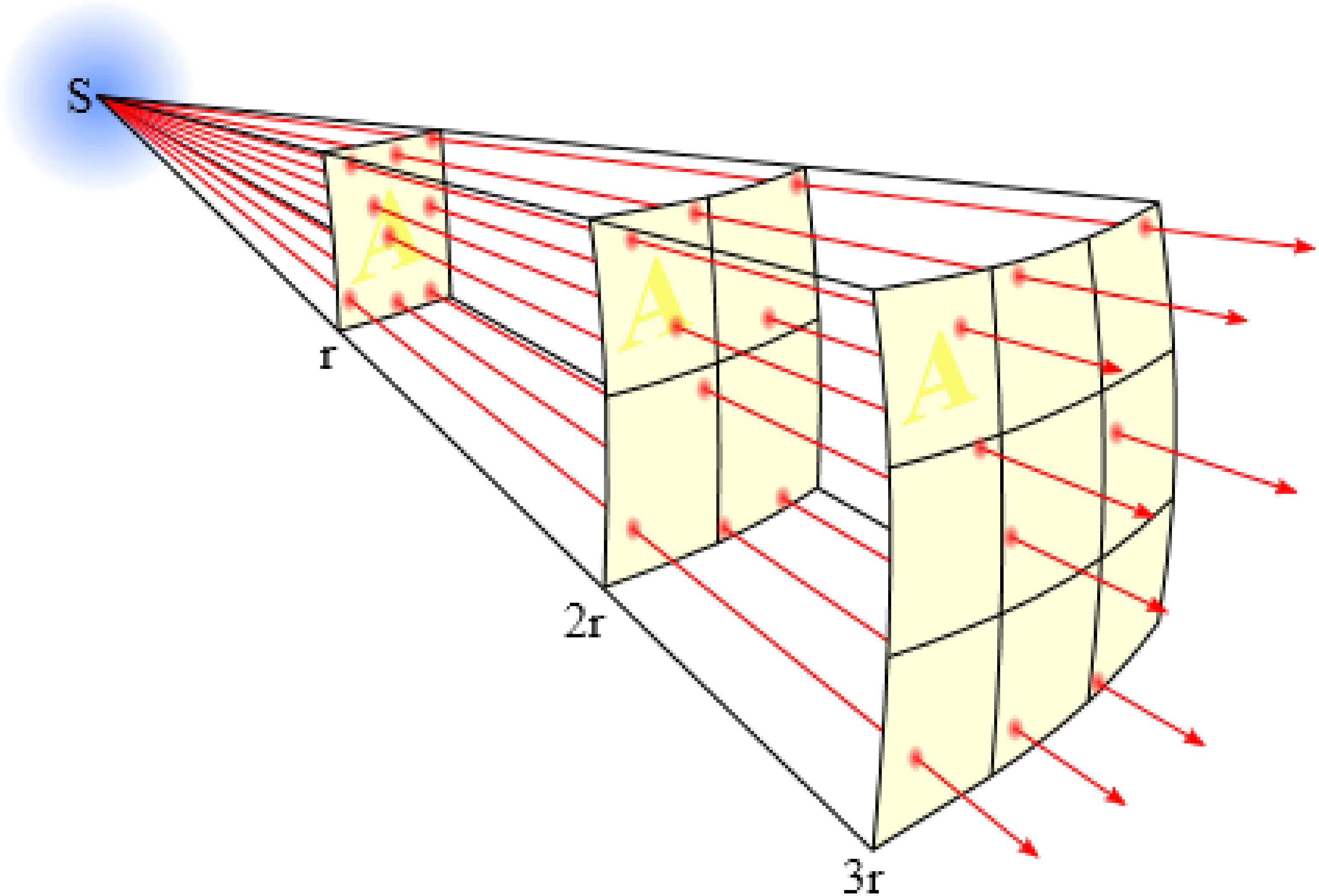
$\frac{2GMr}{R^3}$ at Y and Z, directed **away** from the centre of the Earth

$\frac{GMr}{R^3}$ at T (and similar points) directed **towards** the centre of the Earth


Satellite


60°
30°
0°
-30°
Z


A.
2 Earth radii
59 Earth radii
Common
center of mass
of Earth-Moon System
North
Pole
Earth
Moon
Water level
(distorted by
tide-raising force)
Uniform water level
(theoretical)
Inertial force
Gravitational force
Tide-raising force
B.
Horizontal component
of tide-raising force
60°
30°
0°
30°
60°
To moon
Moon


S
A
A
A
r
2r
3r

What is the size of the bulge?

The interest is in $h$, the difference in the heights of the ocean at Z (or Y) and at T.

For a test mass of mass $m$, this would normally represent a difference in potential energy of $mgh$, where $g$ is the acceleration due to gravity at the surface of the Earth.

But the total potential at T and Z must be the same, otherwise the water would start to flow in order to redistribute itself.

Somehow the tidal effect must have contributed $mgh$ to the total potential at T.

Tidal force on the test mass at Z is $m.\frac{2GMr}{R^3}$, acting **away** from centre of the Earth.

Tidal force on the test mass at the centre of the Earth is zero.
Take the test mass from the centre of the earth to point Z.

The average **assisting** force from the tidal effect would be $\frac{1}{2}.m.\frac{2GMr}{R^3} = \frac{GmMr}{R^3}$, and would apply over a distance $r$.

The work supplied **to** you would be $\frac{GmMr}{R^3}.r = \frac{GmMr^2}{R^3}$

[There is similar result for point Y.]

*If you know calculus.....*

*You can calculate the work supplied* ***to*** *you when going from the centre of the Earth to Z (or Y) rather more rigorously.*

*At a particular point on the path from the centre of the Earth to Z, let* $x$ *be the distance from the centre. The tidal force on a mass* $m$ *there will be* $m.\frac{2GMx}{R^3}$ .

*The elemental work supplied to you when taking the test mass from* $x$ *to* $x + \delta x$ *will be* $\frac{2GmMx}{R^3}.\delta x$

*The total work supplied will be*

$$\sum_{x=0}^{x=r} \frac{2GmMx}{R^3}.\delta x = \frac{2GmM}{R^3}\int_0^r x.dx$$

$$= \frac{2GmM}{R^3}.\left[\frac{x^2}{2}\right]_0^r = \frac{GmMr^2}{R^3}$$

Tidal force on test mass at T is $m.\frac{GMr}{R^3}$, acting **towards** centre of the Earth.

Take the unit mass from centre of the earth to point T.

The average **resisting** force from the tidal effect would be $\frac{1}{2}.m.\frac{GMr}{R^3} = \frac{GmMr}{2R^3}$,
And would apply over a distance $r$.

The work supplied **from** you would be $\frac{GmMr}{2R^3}.r = \frac{GmMr^2}{2R^3}$.

Difference in potential of the mass at T and at Z (or Y) is $\frac{3GmMr^2}{2R^3}$.

This difference in potential drives the water to have a difference in height of h, so:

$$mgh = \frac{3GmMr^2}{2R^3}$$

So: $$gh = \frac{3GMr^2}{2R^3}$$

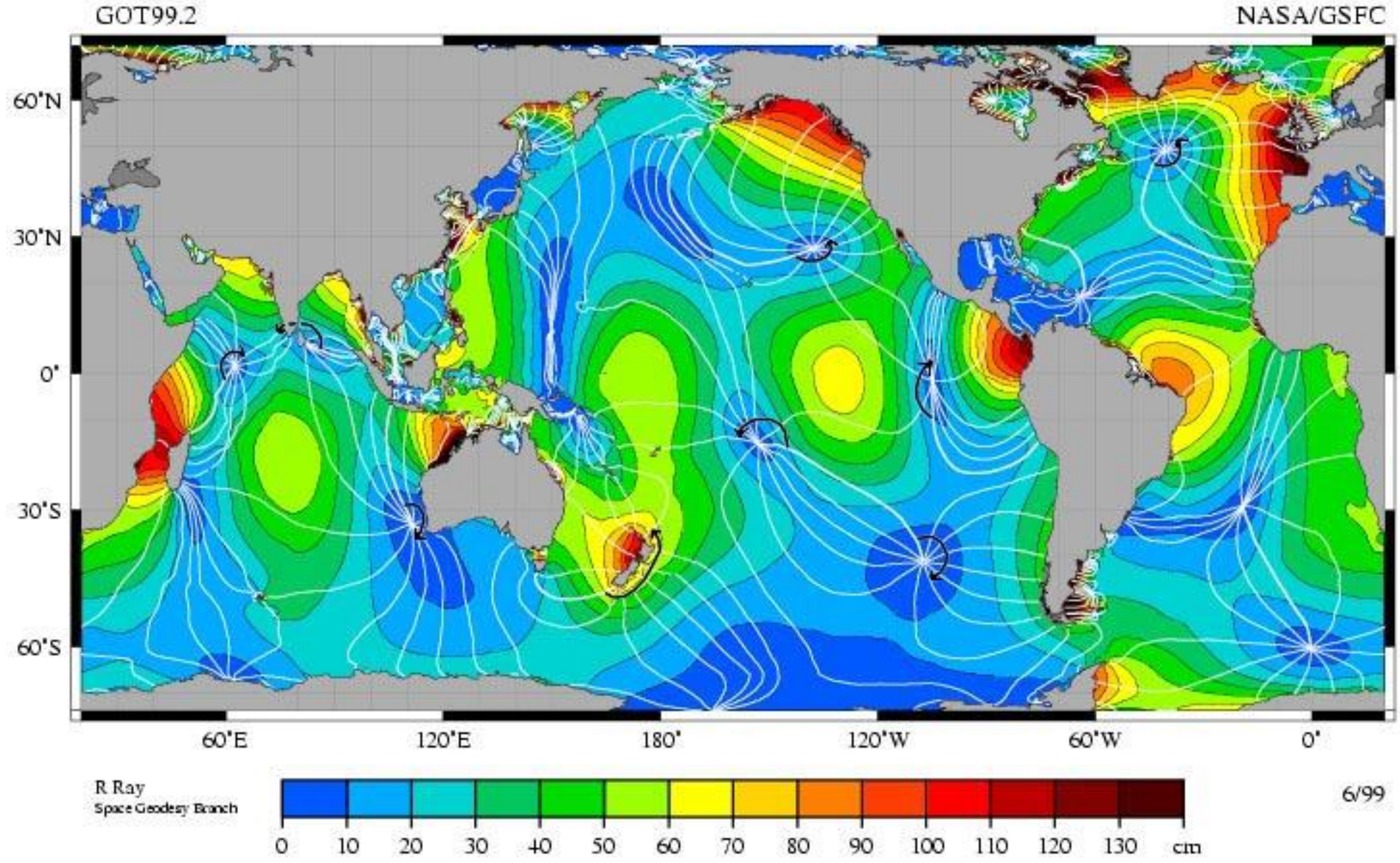
Consider the test mass $m$ at the surface of the Earth.
If the mass of the Earth is $E$, from the inverse square law:

$$mg = \frac{GmE}{r^2}$$

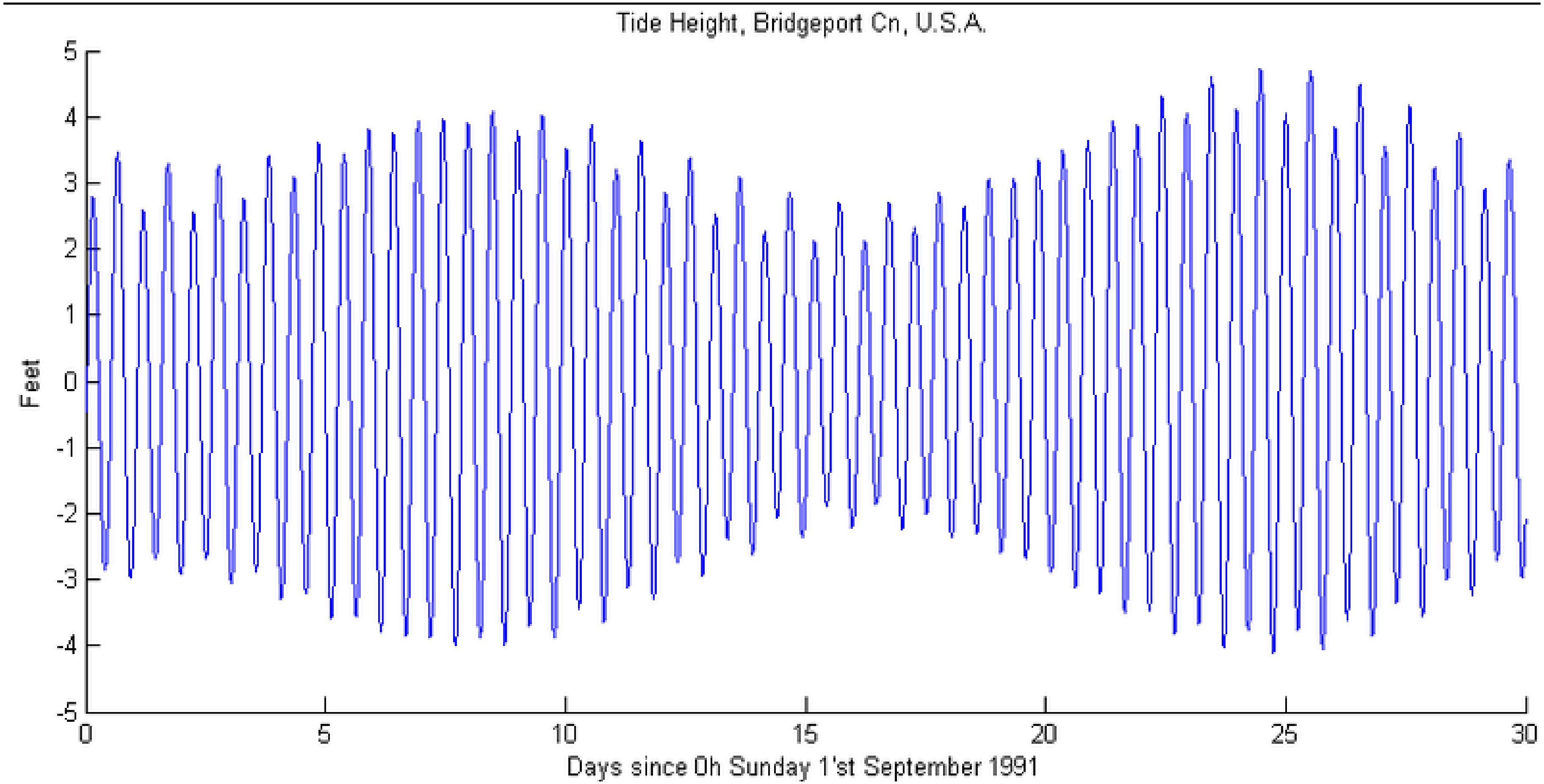
$$\therefore \qquad g = \frac{GE}{r^2}$$

Substitute for $g$ and rearrange:

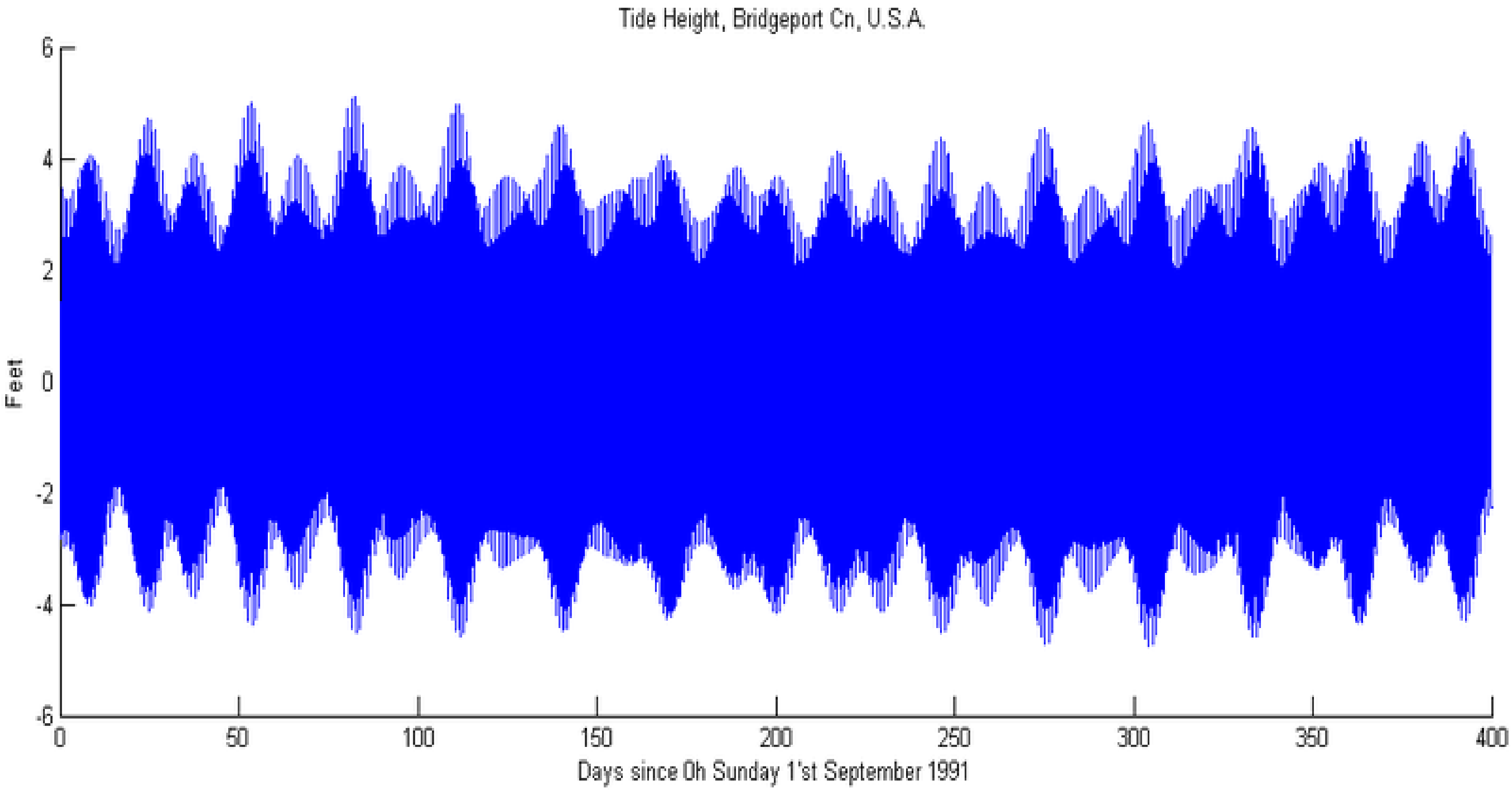
$$h = \frac{3}{2} . \left(\frac{M}{E}\right) . \left(\frac{r}{R}\right)^3 . r$$

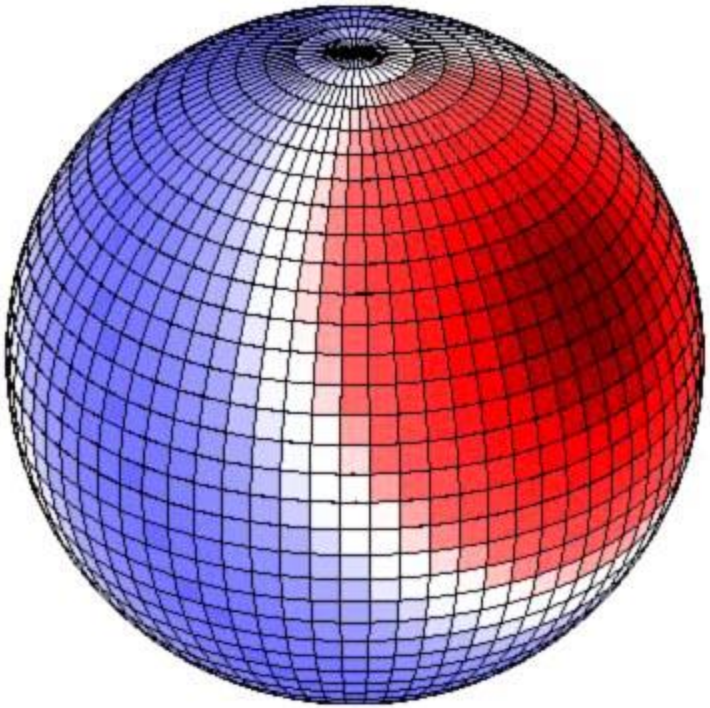


GOT99.2
NASA/GSFC
60°N
30°N
0°
30°S
60°S
60°E
120°E
180°
120°W
60°W
0°
R Ray
Space Geodesy Branch
0
10
20
30
40
50
60
70
80
90
100
110
120
130
cm
6/99

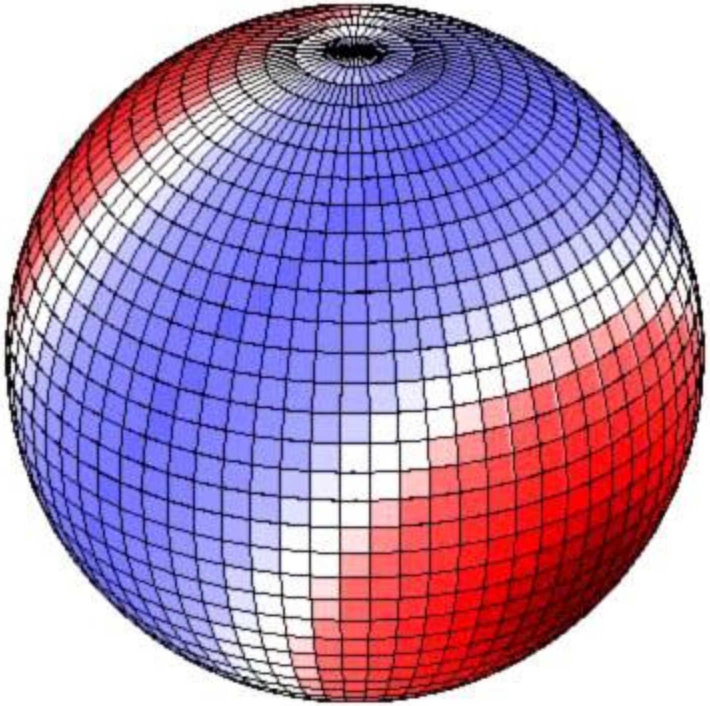


Tide Height, Bridgeport Cn, U.S.A.
Feet
Days since 0h Sunday 1'st September 1991
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
5
10
15
20
25
30

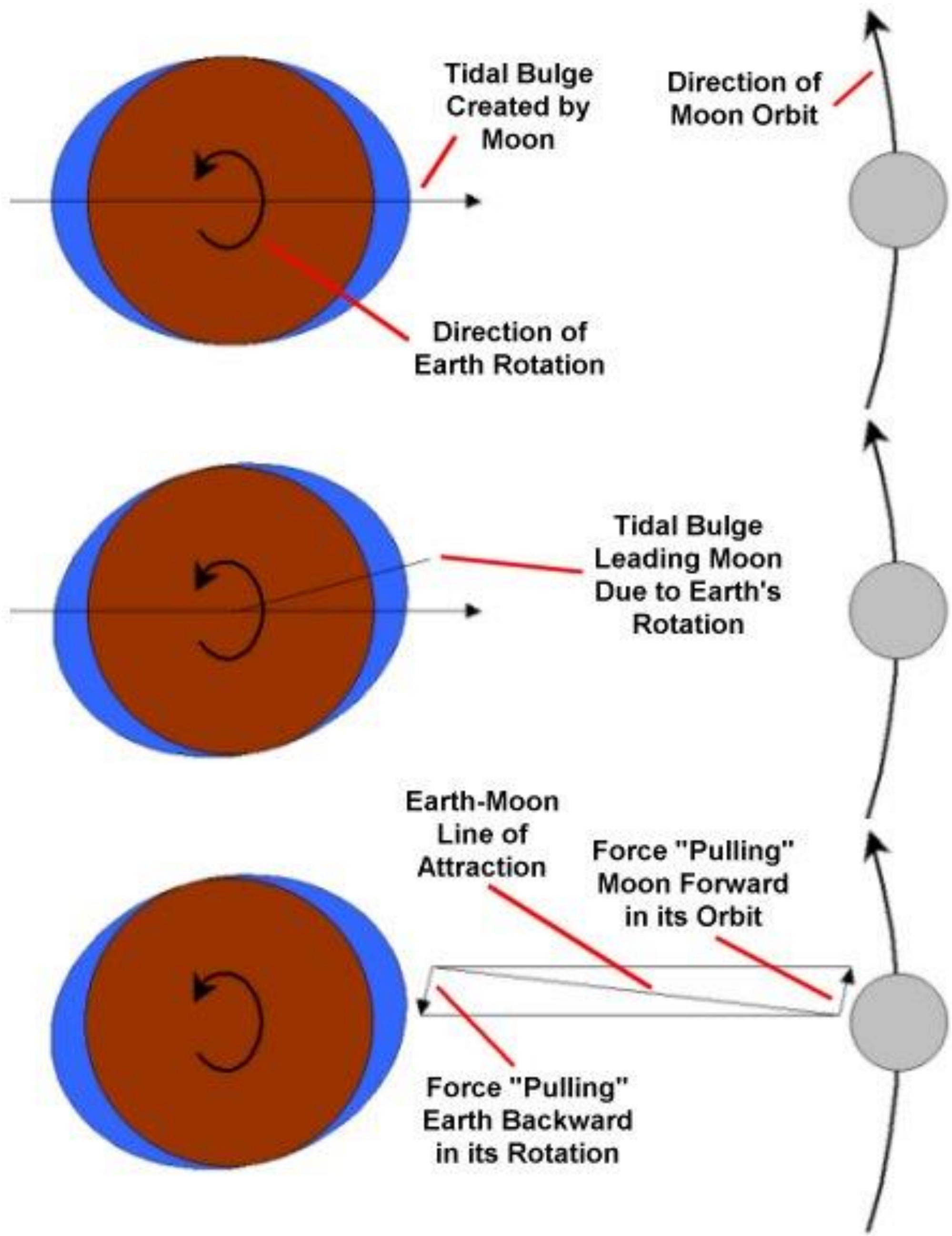

Tide Height, Bridgeport Cn, U.S.A.
Feet
6
4
2
0
-2
-4
-6
0
50
100
150
200
250
300
350
400
Days since 0h Sunday 1'st September 1991



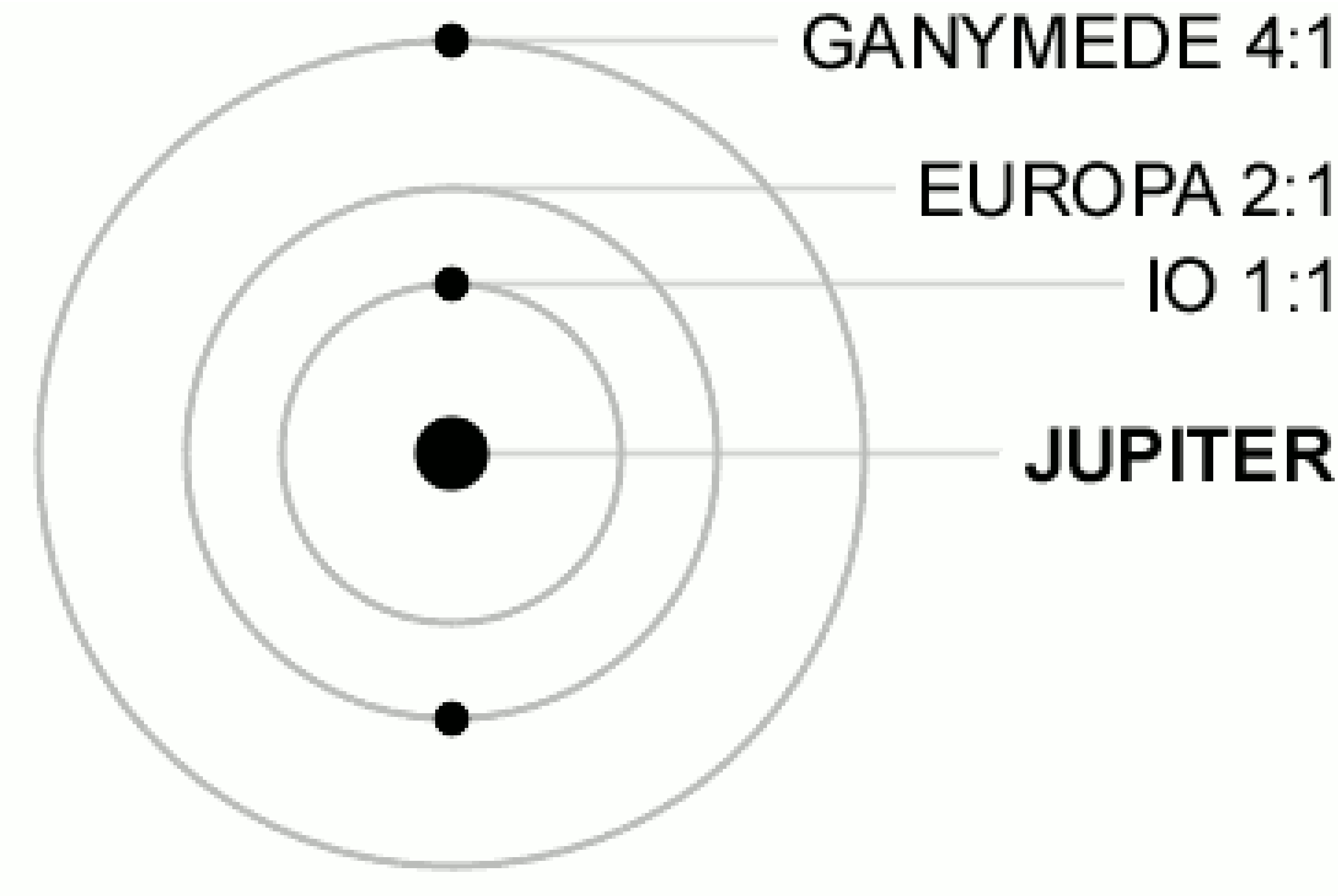
Lunar tidal forcing: this depicts the Moon directly over 30° N (or 30° S) viewed from above the Northern Hemisphere

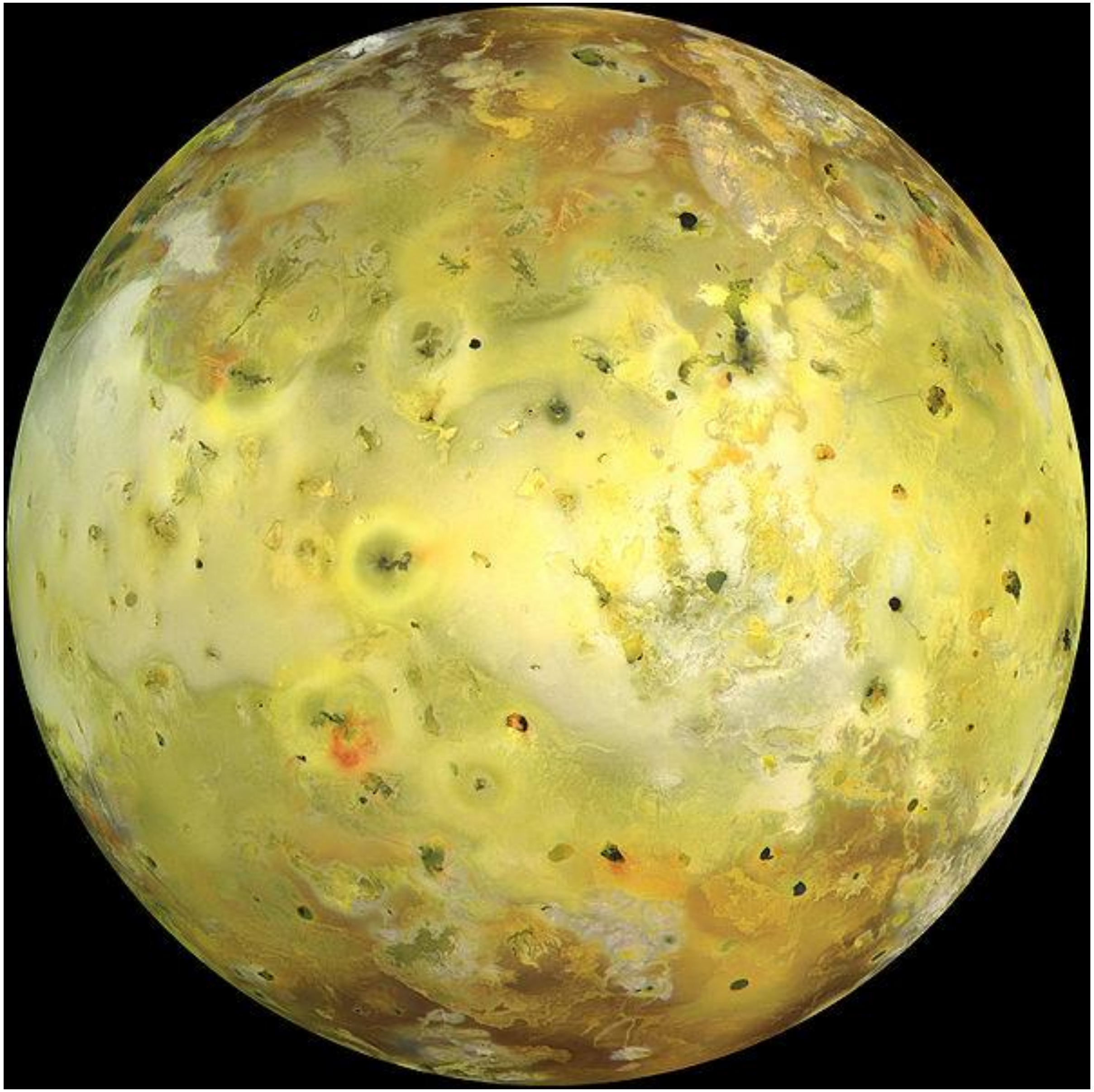


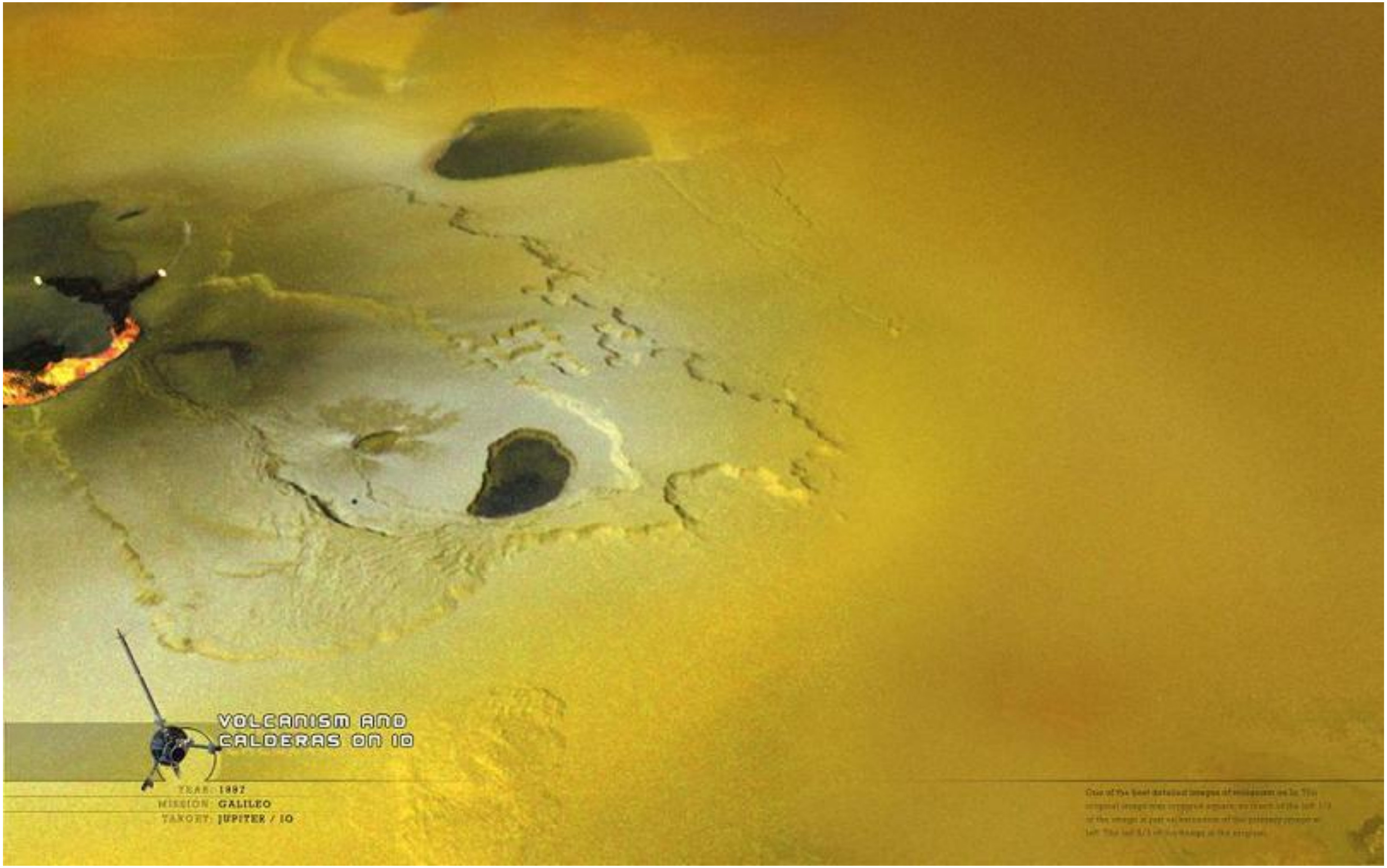
This view shows same forcing 180° from the other view. Viewed from above the Northern Hemisphere. Red up, blue down

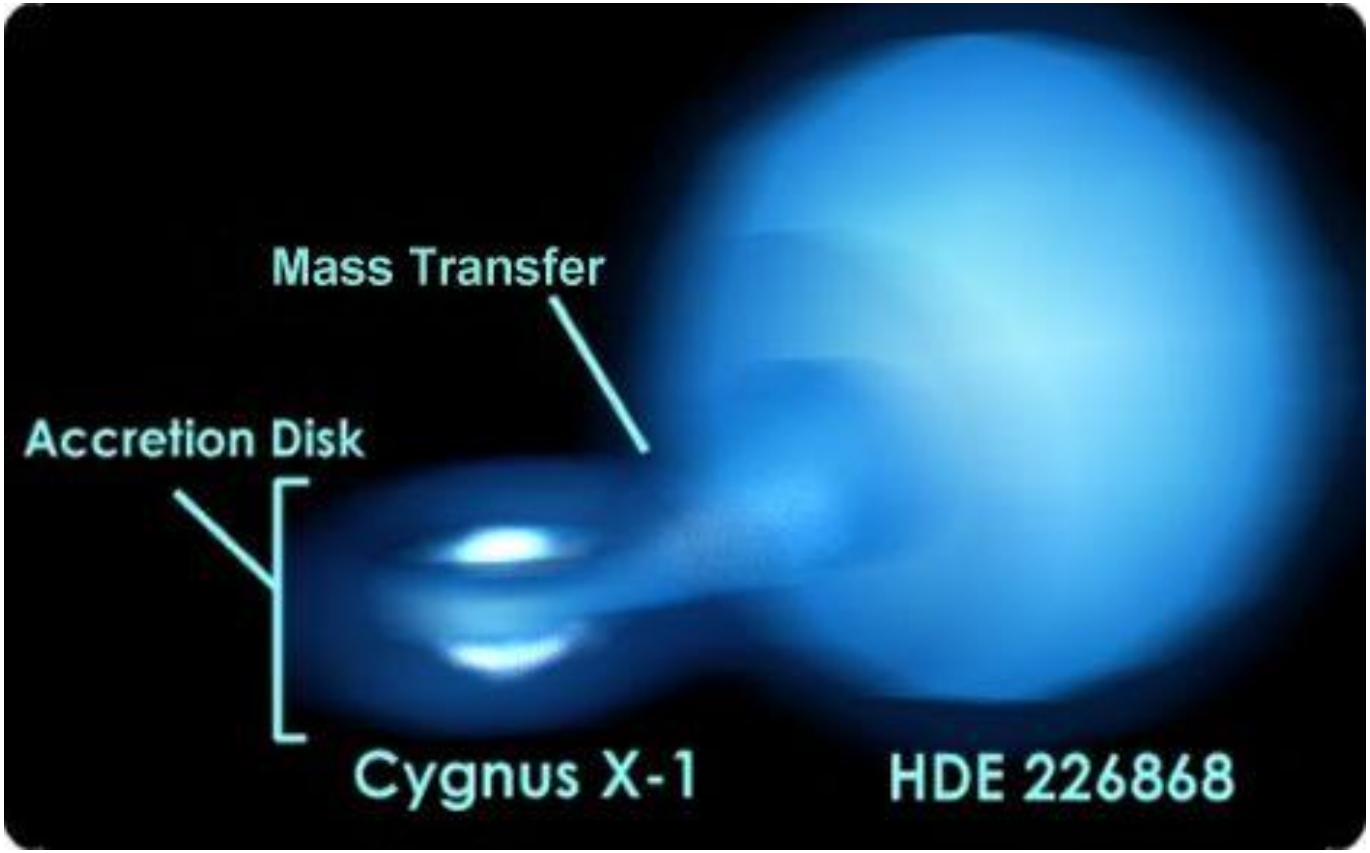


Tidal Bulge Created by Moon
Direction of Moon Orbit
Direction of Earth Rotation
Tidal Bulge Leading Moon Due to Earth's Rotation
Earth-Moon Line of Attraction
Force "Pulling" Moon Forward in its Orbit
Force "Pulling" Earth Backward in its Rotation


GANYMEDE 4:1
EUROPA 2:1
IO 1:1
JUPITER



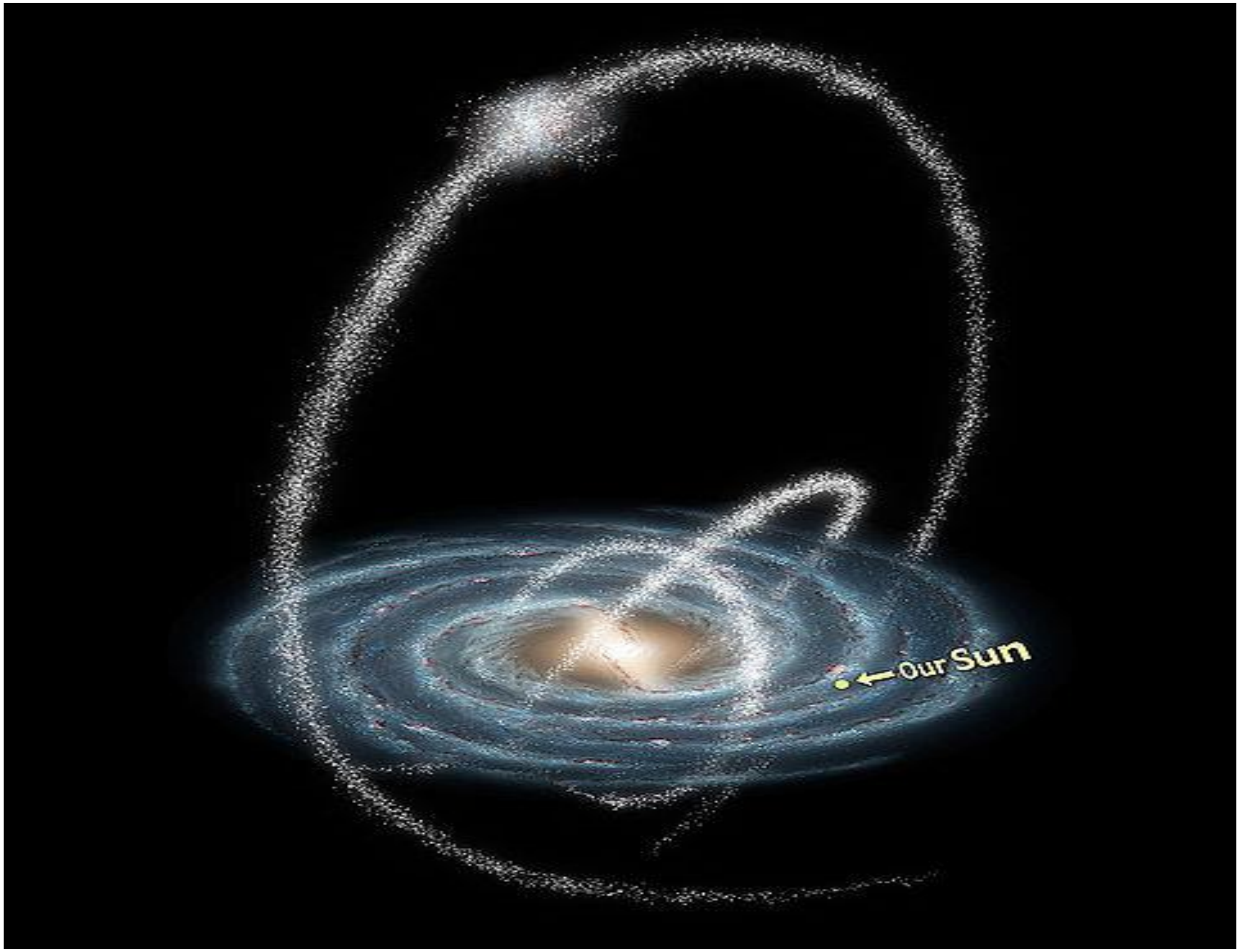

VOLCANISM AND
CALDERAS ON IO
YEAR: 1997
MISSION: GALILEO
TARGET: JUPITER / IO




In 1972, an invisible X-ray source from the constellation **Cygnus** was first detected by X-ray observatory **Uhuru** (Swahili for "freedom") and named **Cygnus X-1**. The X-ray source was found to orbit every 5.6 days around its companion **HDE 226868**, a blue supergiant with 30 solar masses.

Why was Cygnus X-1 considered a black hole? For starters, Cygnus X-1 flickers at less than a thousandth of a second bursts. For an object to flicker, light must travel all the way across its surface. So if light travels 300 kilometres per thousandth of a second, that must mean that Cygnus X-1 is much smaller than our planet.

Second, HDE 226868's spectral lines wobble because of the gravitational pull of Cygnus X-1. In order for that to be possible, scientists estimate that Cygnus X-1 must have at least 7 or more solar masses, putting its mass way beyond the *Oppenheimer-Volkoff Limit* for a neutron star.




Stellar streams in the Milky Way, discovered in 2007